Physics III
ISI B.Math
Midterm Exam : September 7 ,2015

Total Marks: 60
Time : 3 hours
Answer all questions

1. (**Marks: 8 + 4**)

(a) The inside of a spherical metal shell (inner radius R_1 and outer radius R_2) is filled with space charge of uniform charge density ρ. Find the potential at the centre. Find the electrostatic energy of the system.

(b) What is the volume charge density $\rho(\mathbf{r})$ of an electric dipole consisting of a point charge $-q$ at the origin and a point charge $+q$ at $\mathbf{a}$? If $\mathbf{E}(\mathbf{r})$ is the electric field due to this dipole, what is $\nabla \times \mathbf{E}$?

2. (**Marks: 12**)

An infinitely long metal pipe of square cross section (side a) is grounded, but one end , at $x = 0$ is maintained at a constant potential V_0. Show that the potential inside the pipe is given by

$$V(x, y, z) = \frac{16V_0}{\pi^2} \sum_{n,m=1,3,5\cdots}^{\infty} \frac{1}{nm} e^{-\frac{\pi}{a}\sqrt{n^2+m^2}x} \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{m\pi z}{a}\right)$$

3. (**Marks: 4 + 4 +4**)

Charges $+q$, $-q$ lie at the points $(x, y, z) = (a, 0, a), (-a, 0, a)$ above a grounded conducting plane at $z = 0$. Find

(a) The total force on the charge $+q$.

(b) The work done against the electrostatic forces in assembling this system of charges.

(c) The surface charge density at the point $(a, 0, 0)$.

4. (**Marks: 6 + 4 + 2**)

A metal sphere of radius a carries a charge Q. It is surrounded out to radius b by a linear dielectric material of permittivity ϵ.

(a) Find the potential at the centre (relative to infinity).

(b) Find the bound volume charge ρ_b within the dielectric and the bound surface charge σ_b on the inner and outer surfaces of the dielectric.

(c) Are $\nabla \times \mathbf{D}$ and $\nabla \times \mathbf{E}$ both zero everywhere in the above problem ? Justify your answer briefly.

5. (**Marks: 4 + 3 + 5**)

(a) Show that if a particle of charge q and mass m moves in a time independent electric field $\mathbf{E} = -\nabla\phi(x, y, z)$ and *any* magnetic field, then the energy $\frac{1}{2}mv^2 + q\phi$ is a constant, where v is the magnitude of the velocity of the particle.

(b) Explain why the continuity equation $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$ where $\mathbf{J}$ is the volume current density and ρ is the volume charge density , is a mathematical statement of the local conservation of charge.

(c) Find the magnetic field at a point P on the axis of a tightly wound solenoid consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I. Express your answer in terms of θ_1 and θ_2, where θ_1 is the angle made by the axis with the line joining P with the closer end of the solenoid and θ_2 is the corresponding angle made with the farther end. Consider the turns to be essentially circular.